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13. ABSTRACT (Maximum 200 words)

Array processing technology is expected to be a key element in communication systems designed for the crowded and hostile environment of the future battlefield. While advanced array processing techniques have been under development for some time, their practical use has been very limited. This project addressed some of the issues which need to be resolved for a successful transition of these promising techniques from theory into practice. The main problem which was studied was that of finding the directions of multiple co-channel transmitters from measurements collected by an antenna array. Two key issues related to high-resolution direction finding were addressed: effects of system calibration errors, and effects of correlation between the received signals due to multipath propagation. A number of useful theoretical performance analysis results were derived, and computationally efficient direction estimation algorithms were developed. These results include: self-calibration techniques for antenna arrays, sensitivity analysis for high-resolution direction finding, extensions of the root-MUSIC algorithm to arbitrary arrays and to arrays with polarization diversity, and new techniques for direction finding in the presence of multipath based on array interpolation.

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Benjamin Friedlander

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Forward

This report summarizes the work performed on the project "Advanced Techniques for Array Processing" under Contract No. DAAL03-89-C-0007, during the period from 1 March 1989 to 30 April 1991. The project considered the problem of finding the directions of multiple co-channel transmitters from measurements collected by an antenna array. We addressed two key issues related to high-resolution direction finding: effects of system calibration errors, and effects of correlation between the received signals due to multipath propagation. This final report provides a brief overview of the results, deferring the details to previous project publications.

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1. Statement of the Problem

Reliable communications in the crowded and hostile environment of the future battlefield is a difficult and challenging problem. Array processing technology must be a key element in any system designed to face these challenges. While some of the technologies needed for operating in this difficult environment, such as spread spectrum communications and advanced modulation and coding techniques, are fairly mature and well developed, array processing is a relatively new technology and its potential for providing significant gains in system performance has not been fully realized yet. Research and development efforts in the area of array processing are, therefore, expected to have relatively high payoff during the next decade.

While advanced array processing techniques have been discussed in the literature during the past ten years, the practical implementation of these techniques is still in an early stage of development. Various experimental systems using techniques such as MUSIC [41] are now in existence, but as far as we know, at this time there are no fielded systems employing high-resolution array processing algorithms.

In this project we addressed some of the many open issues which need to be resolved for a successful transition of advanced array processing techniques from theory into practice. In particular, we considered the problem of finding the directions of multiple co-channel transmitters from measurements collected by an antenna array. We addressed two key issues related to high-resolution direction finding: effects of system calibration errors, and effects of correlation between the received signals due to multipath propagation.

2. Summary of the Key Results

The results of this work are summarized in 40 project publications which are listed in section 3. Here we present only a brief overview of this work. References [1]–[40] correspond to the project publications listed in section 3, while references [41]–[48] are listed in the bibliography section at the end of the report.

2.1 Calibration Issues

A focal point for our research has been the question: how to apply advanced high-resolution array processing techniques in the presence of various uncertainties in the array parameters. These uncertainties can take on many different forms: imprecisely known location of the array elements, unknown variations in the gains and phases of individual elements, and other deviations between the model on which these techniques are based, and reality.

The problem outlined above is of considerable practical importance. The accurate calibration of the array, required in order to apply "standard" high-resolution techniques, is very costly. Furthermore, in many situations it is practically impossible to maintain the array calibration. Consider for example a mobile communication antenna array which needs to be dismantled and reassembled in the field, its calibration changing each time it is moved. Or consider a conformal array mounted on the wing of an aircraft, where mechanical vibrations constantly change the shape of the array.

To address these problems, which are a major obstacle on the road to a practical application of these promising high-resolution techniques, we have developed a number of techniques to self-calibrate the array using "signals-of-opportunity". We were able to show that it is possible to simultaneously estimate unknown source locations and the unknown array parameters, under some reasonable assumptions [3, 6, 10, 11, 18, 39, 26]. It is important to note that this self-calibration procedure does not require that we know the directions of the calibrating sources.

More recently, we performed an extensive sensitivity analysis of direction finding techniques based on the covariance matrix of the received signals [12, 13, 25]. Using the results of this analysis we can now quantify the effect of various types of calibration errors on the accuracy of the direction-of-arrival (DOA) estimates, and on the ability of the array to resolve closely spaced sources. These results appear to be very useful for the proper design of high-performance direction finding systems. These results also indicate clearly the extreme ill-conditioning of the problem of resolving multiple co-channel signals which are seperated by less than a beamwidth [19].

Our work on the sensitivity issue and the self-calibration problem appears to be the first systematic attempt to analyze and improve the robustness of high-resolution direction finding techniques in the presence of various system errors.

2.2 Direction Finding for Correlated Signals

Direction finding in the presence of coherent multipath is a difficult problem. The multipath causes an apparent distortion of the wavefront of the propagating signal which can introduce large errors into the DOA estimates produced by conventional direction finding systems. High-resolution direction finding systems attempt to alleviate this problem by resolving the components of the multipath arrivals and estimating their DOAs. However, there are various difficulties associated with this procedure as discussed below.

In our work we attempted to address this difficult problem from several points of view: (i) we studied the limits on direction finding accuracy in the presence of multipath; (ii) we developed some practical direction finding algorithms based on the eigenstructure of the covariance matrix of the vector of signals received by the array; (iii) we investigated the potential advantages of using arrays with polarization diversity to alleviate the multipath problem. This work is summarized briefly in the following paragraphs:

Bounds on Direction Finding Accuracy

In [31], [36] we studied the effects of correlation on DOA estimation accuracy by evaluating the corresponding Cramer-Rao bound. We started by deriving the Cramer-Rao bound for Gaussian signals with an unknown covariance matrix (which provides as a by product the CRB for Gaussian signals with a known covariance matrix). We were able to obtain compact, *closed form* expressions for the Fisher Information Matrix associated with the joint estimation of the DOAs, the signal covariance matrix, and the noise variance.

By studying the accuracy of the DOA estimates (as predicted by the CRB) as function of the magnitude and *phase* of the correlation coefficients, we gained insights into the effects of correlation on direction finding performance. As expected, DOA accuracy decreases as the magnitude of the correlation coefficient increases. We have also observed that the *phase* of the correlation coefficient can have a strong effect on DOA accuracy, especially for small aperture arrays. In a multipath environment, the phase of the correlation coefficient is a highly variable, unpredictable quantity, determined by the difference between the propagation delays in the direct and secondary paths. Thus, the best achievable performance of a direction finding system operating in a multipath environment may vary significantly from time to time, due to fluctuations in the correlation phase.

Direction Finding Algorithms

A number of direction finding algorithms for correlated signals have been proposed. These include: spatial smoothing algorithms based on the eigendecomposition of the covariance matrix of the received signals [42, 43]; and algorithms based on maximum likelihood estimation [44, 45].

Direction finding techniques based on the eigen-decomposition of the covariance matrix of the vector of signals received by an array of sensors, have received considerable attention in recent years. Their main advantage over maximum likelihood based techniques, is their relative computational simplicity. The spatial smoothing algorithm involves a one dimensional search, compared to the multi-dimensional search inherent in the maximum likelihood technique. Using the root-MUSIC approach avoids the search completely (or rather, replaces it with a root solving algorithm), reducing even further the computational complexity of the direction finding algorithm. The main drawback of the spatial smoothing technique is that its use is restricted to linear uniformly spaced arrays.

In [33, 37] we were able to generalize the spatial smoothing technique to *arbitrary* array geometries by using an interpolated array approach. This approach is based on estimating the outputs of a "virtual array" from the outputs of the real array. These outputs are obtained by a straightforward linear interpolation technique, with interpolator coefficients selected so as to minimize the interpolation error for a signal arriving from a given sector (*i.e.* a range of bearing angles). Different sets of interpolator coefficients will be used to provide good estimates for different sectors. The size of the sector will be chosen to give sufficiently good estimates of the virtual array outputs. The design of the interpolator needs to be performed only once, off-line.

Given measurements of the real array outputs, the interpolator can be used in principle to compute the outputs of the virtual array. (more precisely, several sets of outputs are computed, one per sector). Any direction finding technique can then be applied to the outputs of the virtual rather than the real array. Actually, it is not necessary to compute the outputs of the virtual array, since it is possible to compute directly the sample covariance matrix corresponding to these outputs. The techniques considered here require only the sample covariance matrix, not the "raw" data.

If the interpolated array is linear and uniformly spaced, the direction of arrival estimates can be computed by using the spatial smoothing method in its root-MUSIC version, which is computationally simple and is more robust to certain modeling errors.

In [32, 34] we carried out a performance analysis for the interpolated spatial smoothing algorithm. Closed form expressions for the covariance matrix of the DOA estimation errors are derived using a perturbation analysis. Evaluating these expressions for specific cases and comparing them to the Cramer Rao lower bound for the DOA estimates, provides insight into the statistical efficiency of this algorithm. The formulas for the error covariance are quite general, and can be specialized to provide results for other DOA estimation algorithms as well. The performance analysis shows that the spatial smoothing estimator (interpolated or not) is not statistically efficient, but it provides acceptable performance under a wide range of operating conditions. The interpolated spatial smoothing algorithms were also tested by Monte-Carlo simulations and was shown to perform well for different test cases. These simulations confirmed the analytical performance predictions.

In summary, the proposed technique combines the computational efficiency associated with eigen-structure based techniques such as root-MUSIC, with good estimation accuracy. It appears to be a very practical approach to the problem of DOA estimation in the presence of multipath.

Direction Finding with Diversely Polarized Arrays

Antenna arrays with diverse polarization have some inherent advantages over uniformly polarized arrays, since they have the capability of separating signals based on their polarization characteristics. Polarization diversity has been used quite successfully in radar systems to improve clutter rejection and to discriminate between different types of targets. Diversely polarized arrays are also useful in the context of communications systems.

One would expect that polarization diversity would be quite useful in high-resolution direction finding systems as well. It is, therefore, somewhat surprising that very little attention has been paid to this issue in the extensive literature on high-resolution array processing. In fact, with the exception of [46], [47] which present the extension of MUSIC and Maximum Likelihood to diversely polarized arrays, there seem to be no results on high-resolution direction finding with these arrays.

Recently, we developed a computationally efficient algorithm for the simultaneous estimation of the DOAs and polarizations of diversely polarized signals [48]. The algorithm is based on polynomial rooting and therefore it exhibits enhanced resolution and reduced computational load (similar to that of the root-MUSIC algorithm which is used in the context of uniformly polarized signals). Since rooting methods can only be applied to certain array configurations, we use linear interpolation in order to extend the technique to a larger class of arrays. Extensive computer simulations were used to demonstrate the effectiveness of the new technique.

In [15, 35] we studied the performance of direction finding systems employing diversely polarized arrays, in a more general setting. In particular, we were interested in comparing the performance of arrays with and without polarization diversity, to try and quantify the performance advantages which can be achieved by using polarization. Our study focused on the statistical accuracy of the direction estimates, using the Cramer Rao Bound (CRB) for the joint estimation of the directions of arrival, polarization parameters, signal covariance matrix, and the noise variance. We were able to develop compact closed form formulas which show the dependence of the bound on various system parameters.

The CRB was derived under the assumption that the signal parameters, namely the polarization parameters and signal covariance matrix (as well as the variance of the measurement noise) are *unknown* and need to be estimated as part of the direction finding process. This bound is more realistic than the one which assumes that the signal (and noise) parameters are known, since in practice neither the signal polarization nor its correlation properties are known *a priori*. Furthermore, DOA estimation algorithms such as [46, 47], do not make use of prior information regarding signal parameters. Thus, their performance is likely to be closer to that of the CRB which assumes no knowledge of signal parameters, than to the CRB which assumes such knowledge.

We evaluated the CRB for some selected examples and plotted the standard deviation of the DOA estimates as a function of various system parameters. This numerical study indicates that in general, polarization diversity improves the accuracy of direction finding systems in the presence of signal correlation. In particular, we note that the performance of arrays which do not make use of polarization diversity, is very sensitive to the *phase* of the signal correlation, which is a highly unpredictable and uncontrollable parameter. Arrays which use polarization diversity appear to be much less sensitive to this parameter.

In summary, the use of polarization diversity is potentially very useful for direction finding in the presence of multipath, provided that there is a significant difference between the polarizations of the direct and secondary (reflected) signals.

2.3 Using High Order Statistics

Another area of research undertaken in this project was the development and performance analysis of parameter estimation algorithms based on high-order statistics, and their application to communications problems. Some basic theoretical results on this topic were presented in [2, 7, 23].

High-resolution direction finding techniques have been based on the covariance matrix of the received signals, *i.e.*, on second order statistics. This raises the intriguing question of whether the higher order statistics of the signals contain useful information about the signal DOAs, and if so, how could this information be exploited. In [20, 29] we made a very preliminary attempt to address this problem and developed a MUSIC-like algorithm which exploits fourth-order cumulants to provide DOA estimates. In future work we plan to investigate the potential of high-order techniques for direction finding in the presence of multipath.

In [14, 21, 27, 30] we presented some preliminary results on adaptive channel equalization algorithms based on high-order statistics. This work seems to indicate that improved equalization performance may be achieved in some situations by optimal utilization of the information contained in the high-order statistics of the signal.

3. List of Project Publications

3.1 Journal Publications

1. B. Friedlander and B. Porat, "Detection of Transient Signals by the Gabor Representation," *IEEE Trans. Acoustics, Speech and Signal Processing*, vol. ASSP-37, no. 2, pp. 169–180, February 1989.

2. B. Friedlander and B. Porat, "Adaptive IIR Filtering Based on High-Order Stastistics." *IEEE Trans. Acoustics, Speech and Signal Processing*, vol. 37, no. 4, pp. 485–495, April 1989.

3. B. Friedlander and A. J. Weiss, "Self Calibration in Array Processing," Ch. 10 in *Advances in Spectrum Analysis and Array Processing, Vol. II*, S. Haykin, Ed., Prentice Hall, Englewood Cliffs, 1991.

4. B. Friedlander and B. Porat, "The Square-Root Overdetermined Recursive Instrumental Variable Algorithm," *IEEE Transactions on Automatic Control*, vol. 34, No. 6, pp. 656–658, June 1989.

5. B. Friedlander and B. Porat, "Performance analysis of a null-steering algorithm," *IEEE Trans. Acoustics, Speech and Signal Processing*, vol. 37, no. 4, pp. 461–466, April 1989.

6. A. J. Weiss and B. Friedlander, "Array shape calibration using sources in unknown locations: A maximum likelihood approach," *IEEE Trans. Acoustics, Speech and Signal Processing*, vol. 37, no. 12, pp. 1958–1966, December 1989.

7. B. Porat and B. Friedlander, "Asymptotically Optimal Estimation of MA and ARMA Parameters of Non-Gaussian Processes from High Order Moments", *IEEE Trans. Automatic Control*, special issue on High Order Statistics in System Theory and Signal Processing, vol. AC-35, No. 1, pp. 27-35, January 1990.

8. T. Söderström, P. Stoica, and B. Friedlander, "An indirect Prediction Error Method for System Identification," *Automatica*, vol. 27, No. 1, pp. 183-188, 1991.

9. B. Friedlander and B. Porat, "The exact Cramer-Rao bound for Gaussian autoregressive processes," *IEEE Trans. Aerospace Electron. Systems*, vol. AES-25, No. 1, pp. 3–8, January, 1989.

10. B. Friedlander, and A. J. Weiss, "Eigenstructure methods for direction finding with sensor gain and phase uncertainties," *J. Circuits, Systems and Signal Processing*, vol. 9, no. 3, pp. 271–300, 1990.

11. B. Friedlander and A. J. Weiss "Direction Finding in the Presence of Mutual Coupling," *IEEE Trans. Antennas and Propagation* vol. AP-39, No. 3, pp. 273–284, March 1991.

12. B. Friedlander, "A Sensitivity Analysis of the Maximum Likelihood Direction Finding Algorithm," *IEEE Transactions on Aerospace Electronic Systems*, vol. AES-25, no. 6, pp. 953–968, November 1990.

13. B. Friedlander, "A Sensitivity Analysis of the MUSIC Algorithm," *IEEE Transactions on Acoustics, Speech and Signal Processing*, vol. 38, no. 10, pp. 1740–1751, October 1990.

14. B. Friedlander and B. Porat, "Adaptive Channel Equalization Based on High Order Moments," *IEEE Trans. Acoustics, Speech and Signal processing*, vol. 39, no. 2, pp. 502–526, February, 1991.

15. A. J. Weiss and B. Friedlander, "Performance Analysis of Diversely Polarized Antenna Arrays," *IEEE Transactions on Acoustics, Speech and Signal Processing.*

16. B. Friedlander, "Transient Signal Detection Techniques," *U. S. Navy J. Underwater Acoustics.*

17. B. Friedlander and B. Porat, "The Square-Root Overdetermined Recursive Instrumental Variable Algorithm," in *Mathematics in Signal Processing II*, Clarendon Press, Oxford, 1990.

18. A. J. Weiss and B. Friedlander, "Array Shape Calibration via Eigenstructure Methods," *J. Signal Processing.*

19. B. Friedlander an A. J. Weiss, "On the Number of Signals Whose Directions Can Be Estimated by an Array," *IEEE Transactions on Acoustics, Speech and Signal Processing.*

20. B. Porat and B. Friedlander, "Direction Finding Techniques Based on High Order Statistics," *IEEE Transactions on Acoustics, Speech and Signal Processing*.

3.2 Conference Publications

21. B. Friedlander, and B.Porat, "Adaptive Channel Equalization Based on High-Order Statistics," Intl. Conference on Acoustics, Speech and Signal Processing, April, 1989.

22. B. Friedlander, "Sensitivity Analysis of High-Resolution Array Processing," Intl. Conference on Acoustics, Speech and Signal Processing, April, 1989.

23. B. Friedlander and B.Porat, "Asymptotically Optimal Estimation of MA and ARMA Parameters of Non-Gaussian Processes from High-Order Moments," Workshop on Higher-Order Spectral Analysis, Vail, Colorado, June 28–30, 1989.

24. B. Friedlander, " Sensitivity Analysis of High-Resolution Array Processing Algorithms," Workshop on Multi-Dimensional Signal Processing, Pacific Grove, California, September 1989.

25. B. Friedlander, "Sensitivity Analysis of the Maximum Likelihood Direction Finding Algorithm," 23rd Asilomar Conference on Signals, Systems and Computers, Pacific Grove, California, October 30 – November 1, 1989.

26. A. J. Weiss and B. Friedlander, "Array Shape Calibration via Eigenstructure Methods," 23rd Asilomar Conference on Signals, Systems and Computers, Pacific Grove, California, October 30 – November 1, 1989.

27. B. Porat and B. Friedlander, "Performance Analysis of a Blind Channel Equalization Algorithm Based on High Order Moments," 23rd Asilomar Conference on Signals, Systems and Computers, Pacific Grove, California, October 30 – November 1, 1989.

28. B. Friedlander, "Direction Finding with an Interpolated Array," Intl. Conference on Acoustics, Speech and Signal Processing, Albuquerque, New Mexico, April 3–6, 1990.

29. B. Porat and B. Friedlander, "Direction Finding Algorithms Based on High Order Statistics," Intl. Conference on Acoustics, Speech and Signal Processing, Albuquerque, New Mexico, April 3–6, 1990.

30. B. Porat and B. Friedlander, "Adaptive Blind Equalization Using 4-the Order Cumulant Matrices," 24the Asilomar Conference on Signals, Systems and Computers, Pacific Grove, California, November 5 - November 7, 1990.

31. A. J. Weiss and B. Friedlander, "On the Cramer-Rao Bound for Direction Finding for Correlated Signals," 24the Asilomar Conference on Signals, Systems and Computers, Pacific Grove, California, November 5 - November 7, 1990.

32. A. J. Weiss and B. Friedlander, "Performance Analysis of Spatial Smoothing with Interpolated Array," Intl. Conference on Acoustics, Speech and Signal Processing, Toronto, Canada, May 1991.

3.3 Under Review

33. B. Friedlander and A. J. Weiss, "Direction Finding for Correlated Signals Using Spatial Smoothing with an Interpolated Array," *IEEE Trans. ASSP.*

34. A. J. Weiss and B. Friedlander, "Performance Analysis of Spatial Smoothing with Interpolated Array," *IEEE Trans. ASSP.*

35. B. Friedlander and A. J. Weiss, "Performance of Diversely Polarized Antenna Arrays for Correlated Signals," *IEEE Trans. Aerospace Electronic Systems.*

36. A. J. Weiss and B. Friedlander, "On the Cramer Rao Bound for Direction Finding of Correlated Signals," submitted for publication.

37. B. Friedlander, "The Interpolated Root-MUSIC Algorithm for Direction Finding," submitted for publication.

38. B. Friedlander and A. J. Weiss, "Direction Finding for Wideband Signals Using an Interpolated Array," *IEEE Transactions on Acoustics, Speech and Signal Processing.*

39. B. Friedlander and A. J. Weiss, "Performance of Direction Finding Systems with Sensor Gain and Phase Uncertainties," *IEEE Transactions on Acoustics, Speech and Signal Processing.*

40. B. Porat and B. Friedlander, "Blind Equalization Using Fourth Order Cumulants," *IEEE Transactions on Acoustics, Speech and Signal Processing.*

4. List of all Participating Technical Personnel

Dr. Benjamin Friedlander
Dr. Anthony J. Weiss
Dr. Boaz Porat

Bibliography

41. R.O. Schmidt, "Multiple Emmitter Location and Signal Parameter Estimation," in *Proc. RADC Spectrum Estimation Workshop,"* Griffiths AFB, New York, 1979, pp. 243–258. Reprinted in *IEEE Trans. Antennas and Propag.*, vol. AP-34, no. 3, pp. 276–380, March 1986.

42. J.E. Evans, J.R. Johnson, D.F. Sun, "Application of Advanced Signal Processing Techniques to Angle of Arrival Estimation in ATC Navigation and Surveillance System," M.I.T. Lincoln Lab., Lexington, MA, Rep. 582, 1982.

43. T.J. Shan, M. Wax, T. Kailath, "On Spatial Smoothing of Estimation of Coherent Signals," *IEEE Trans. Acoust., Speech, Signal Processing*, vol.ASSP-33, pp. 806–811, Aug. 1985.

44. I. Ziskind and M. Wax, "Maximum Likelihood Localization of Multiple Sources by Alternating Projection," *IEEE Trans. Acoust., Speech, Signal Processing*, vol. ASSP-36, pp. 1553-1560, Oct. 1988.

45. J.F. Bohme, "Estimation of Source Parameters by Maximum Likelihood and Nonlinear Regression", in *Proc. ICASSP* , pp. 7.3.1-7.3.4. Apr. 1984.

46. E.R. Ferrara, T.M. Parks, "Direction Finding with an Array of Antennas Having Diverse Polarizations," *IEEE Trans. on Antennas and Propagation*, vol. AP-31, no. 2, pp. 231–236, March 1983.

47. R. O. Schmidt, "A Signal Subspace Approach to Multiple Emitter Location and Spectral Estimation," Ph.D. Dissertation, Stanford University, Stanford, California, 1981.

48. B. Friedlander and Anthony J. Weiss, "Direction Finding for Diversely Polarized Signals Using an Interpolated Array," in preparation.

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